1 Introduction

This article considers a model of voting where agents are restricted in their ability to communicate to the principal. Observing their private type, the agents choose one of a (small) finite number of signals that they send to the principal. Such communication restrictions are common in democratic electoral systems as well as in allocation problems on the internet. Common to all is that the type space of agents (be it voters or internet servers) is much more complex than what they can or want to communicate to a mechanism. Communication restrictions arise for different reasons:
First, it is costly to communicate the exact position in a large type space. In computing environments, economic decisions involve low-cost computational resources. The communication of even a single-precision integer (this is a number in $0, 1, 2, \ldots, 2^{32} - 1$) might be too costly. A small number of different signals that agents communicate might hence be a desideratum and sometimes even a constraint for mechanism design for the internet.
Furthermore, agents might be hesitant to reveal their type. For example, in income and wealth questionnaires, depending on culture, direct questions for the income of an agent are omitted as these would not be answered—instead, the agents announce to lie in one of a pre-specified list of intervals.
As a third reason, there might be intellectual limits to the agents' perception. If there are 10,000 options, an agent might be overwhelmed by the number of alternatives and in fact only evaluate a few of the alternatives, e.g. the first and last few of them. In addition, informed debate of alternatives could often result in too high opportunity costs.
Therefore, studying a model with restricted communication will yield valuable insights into real-world voting mechanisms.

Nevertheless, much of the mechanism design literature is based on the revelation principle. The revelation principle says that any mechanism can be implemented by the agents revealing their true type. The argument is that for any mechanism with a different communication structure, a "black box" could be introduced that takes the types of the agents and "plays" for them in an optimal way—assuming agents announce their type truthfully. For the reasons given above, the communication of one's type to such a black box is not feasible in many environments. Hence, one cannot consider direct revelation mechanisms in a realistic model of mechanism design with restricted communication, but has to model the mappings from types to messages.

A body of literature studies mechanism design with limited communication and with monetary transfers. In many situations, however, monetary transfers are not feasible: First, transaction costs might be too high to justify a monetary transfer. This might be the case if the economic decision to be taken is one of low stakes, e.g. if the agents decide on low-value computing resources. In other situations, monetary transactions are not used for ethical reasons, as in democratic elections. The present study contributes to the literature on mechanism design *without* monetary transfers.

This work is to the best of our knowledge the first contribution to Dominant Incentive Compatible (DIC) mechanism design with restricted communication and without monetary transfers. Our contributions are threefold:

First-Best In a one-dimensional voting model with quadratic utilities, we give a strong necessary condition for first-best mechanisms under restricted communication. We provide upper and lower bounds on the rate of convergence of average welfare in the size of the message set and show that the range of any first-best mechanism grows with the size of the society.

Second-Best For the single-peaked preference domain, we show that anonymous, dominant strategy implementable, non-wasteful (a new definition we give) mechanisms are exactly *embedded generalised median voting rules*. Contrasting the first-best case, these have a small range. We illustrate that the welfare cost of a communication restriction is strictly higher when taking into account incentive constraints.

Other Preference Restriction We identify two properties (Properties A and B) that preference domains need to satisfy such that the characterisation for single-peaked preferences also holds. We show that any regular, single-crossing, tops-connected (RST) preference domain, among them the quadratic and a continuouus extension of Romer et al. 1975's linear preference domain, satisfies property A and a property only slightly weaker than property B, property B'.

Literature Review

Three branches of literature are particularly important for our work: approximation of mechanisms by simple mechanisms, mechanism design with limited communication with monetary transfers and, from the electrical engineering literature, quantisation and distributed inference.

First, there has been an extensive literature on approximation of mechanisms by simple mechanisms: See McAfee 2002 and Hoppe et al. 2011 for an analysis of the "coarse" matching matching algorithm, Ledyard and Palfrey 2002 for a Bayesian Incentive Compatible (BIC) analysis of two-message voting rules in public goods provision and Gershkov et al.

2017, Section 5, p. 21 for a similar asymptotic result in a DIC setting. For the allocation of a divisible good, Wilson 1989 studies DIC screening where only k pre-defined types ("priority classes") can be communicated by the agents of signals and shows quadratic convergence of welfare in k. Our work differs from this literature in that it does not consider a fixed class of mechanisms with a certain structure that approximate an optimal one, but considers a constrained optimal mechanism subject to a bound on the number of different messages an agent can send.

Furthermore, there is an extensive literature on mechanism design under limited communication with *monetary* transfers. Blumrosen and Feldman 2006 characterises revenue-maximising mechanisms and convergence of revenue in a DIC setting, assuming single-crossingness and multi-linearity of the welfare function, Blumrosen, Nisan, et al. 2007 studies the convergence of revenue in a BIC setting and welfare convergence in a DIC setting and provides a characterisation of resp. optimal mechanisms for a small number of agents resp. goods. In a slightly different model, Bergemann, Shen, Xu, and E. M. Yeh 2011 does a similar analysis with a necessary conditions for optimal mechanisms for any number of agents and one good. Bergemann, Shen, Xu, and E. Yeh 2012 restricts itself to welfare maximisation and provides an analysis for an arbitrary number of goods. Our work differs from this literature in that we do not allow for monetary transfers.

Finally, in the electrical engineering and statistics literature, communication under constraints is an important topic of study. There are two branches of literature particularly connected to the present study: quantisation and distributed inference.
Quantisation is the theory of the optimal approximation of a real-valued (or more general) random variable, the "signal", by a finite number of discrete levels or subject to a bound on the entropy of the discretised random variable. The further is more relevant to our setting. Lloyd 1982 and Max 1960 independently discovered a strong necessary condition for the optimal quantisation of a square-integrable random variable, whose convergence guarantee is the best known for general distributions.
Distributed Inference is the problem of estimating a quantity from observations by different sensors by a principal—the "fusion center". The sensors send quantised messages to the principal. The principal uses a mechanism, the "fusion rule" to determine a quantity from the different messages. By treating the sensors as agents that act strategically, this is a mechanism design problem with restricted communication. Such cases might occur if the sensors are in public. See e.g. Kailkhura et al. 2015 and Venkitasubramaniam et al. 2007 for robustness of distributed inference problems.

2 Model

Agents and Utilities We consider n agents $i = 1, 2, \ldots, n$ that have to make a choice on a continuous value of common interest. Call the (bounded) *set of outcomes* $\mathcal{K}$ and the *set of types* $\mathcal{T}$ which we both assume to be totally ordered by $<$ (e.g. $\mathcal{T} = \mathbb{R}$ and $\mathcal{K}$ a subset of $\mathbb{R}$). The type set $\mathcal{T}$ consists of preference orders, i.e. reflexive, transitive, asymmetric relations over alternatives in $\mathcal{K}$. Write $\mathcal{T}_A \colon \{t|_{A\times A} \mid t \in \mathcal{T}\}$ for the preferences in $\mathcal{T}$ restricted to the set $A \subseteq \mathcal{K}$. Agent $i = 1, 2, \ldots, n$ has a type $t_i \in \mathcal{T}$ privately known to her. By $\tau_{\mathcal{K}}(t) \in \mathcal{K}$ we denote the most preferred alternative of an agent with type $t \in \mathcal{T}$. Utility functions $u^x \colon \mathcal{T} \to \mathbb{R}, \quad t \mapsto u^x(t)$ induce preferences that are not strict. When it is of importance how to resolve indifferences in utility (i.e. $x, x' \in \mathcal{K}$ such that $u^x(t) = u^{x'}(t)$) we will state it.

In the different sections, we study different domains of preferences $\mathcal{T}$. In subsection 3.1 and section 4, we consider quadratic preferences: $\mathcal{T} = \mathcal{K} = [0, 1]$ with utility functions $u^x_{\text{quad.}}(t) = -(t-x)^2$. In subsection 3.2, we consider the set of all single-peaked preferences. A preference relation $t \in \mathcal{T}$ is *single-peaked* if for any $x_1, x_2 \in \mathcal{K}$ such that $x_2 < x_1 < \tau_{\mathcal{K}}(t)$ or $\tau_{\mathcal{K}}(t) > x_2 > x_1$ it holds that $\tau_{\mathcal{K}}(t) \succeq^t x_1 \succeq^t x_2$.

Mechanisms and Strategies A deterministic *indirect* mechanism asks agents to report one message $m \in \mathcal{M}$ where $\mathcal{M}$ is a *message set*, whose cardinality we call k. The mechanism then chooses an alternative from $\mathcal{K}$. Formally, an indirect mechanism is a function $g \colon \mathcal{M}^n \to \mathcal{K}$. In the following, we just write "mechanism" if there is no risk of ambiguity. We assume implicitly that participation in the mechanism is obligatory. We are interested in the case where $\mathcal{M}$ is finite, hence, where the agents cannot report their complete preferences, but their report must be noisy.

We call a mechanism $g \colon \mathcal{M}^n \to \mathcal{K}$ *anonymous* if for any permutation $\pi \in S_n$ and any messages $m_1, m_2, \ldots, m_n \in \mathcal{M}$, $g(m_1, m_2, \ldots, m_n) = g(\pi(m_1), \pi(m_2), \ldots, \pi(m_n))$ holds. Less formally, this is the well-known requirement for a voting system that identity of the voters should be irrelevant for the outcome of the voting system.

A *strategy* for agent i, $i = 1, 2, \ldots, n$ is a mapping from types to reports, formally $s_i \colon \mathcal{K} \to \mathcal{M}$. We stress that we only allow for *pure* strategies. For messages $m, m_1, m_2, \ldots, m_n$ we will write (m, m_{-i}) for $(m_1, m_2, \ldots, m_{i-1}, m, m_{i+1}, \ldots, m_n) \in \mathcal{M}^n$.

A mechanism g is said to be *implementable in dominant strategies* by strategies s_1, s_2, …, s_n if for any $i = 1, 2, \ldots, n$ and any $m_{-i} \in \mathcal{M}^{n-1}$ it holds that $g(s_i(t_i), m_{-i}) \succeq^{t_i} g(m, m_{-i})$. Less formally, $s_i(t_i)$ must be a best response given *any* messages the other agents send.

We assume two *efficiency* requirements, one concerning the mechanisms and one concerning strategies. The first is needed to rule out different strategies yielding an ex-post identical outcome. A mechanism is *non-wasteful* (in the sense that all communication is informative) if

$$\text{for any } m, m' \in \mathcal{M} \text{ there is } m_{-i} \in \mathcal{M}^{n-1} \text{ such that } g(m, m_{-i}) \neq g(m', m_{-i}). \tag{1}$$

The second assumption requires that for the mechanism the size k of the message set is actually needed—For any message, there is a type that plays it. More formally,

$$\text{all } s_i \colon \mathcal{T} \to \mathcal{M} \text{ are surjective.} \tag{2}$$

Type Distribution and Welfare Maximisation We assume that $t_1, t_2, \ldots, t_n \sim F$ are i.i.d. We call $W(F, g) \coloneqq \frac{1}{n}\mathbb{E}\left[\sum_{i=1}^{n} u^{x_i}(g(s_1(t_1), s_2(t_2), \ldots, s_n(t_n)))\right]$ the *(average ex-ante) welfare* of the mechanism g. A mechanism that is welfare maximising is said to be *first-best*. A mechanism that is ex-ante welfare maximising among all anonymous, non-wasteful mechanisms that are dominant strategy implementable by surjective strategies is called *second-best*. We use this non-standard terminology for the sake of brevity.

3 Characterisation and Welfare Analysis

3.1 First-Best

In this section, we consider quadratic utility functions $u^{t_i}(x) = -(t-x)^2$ and $\mathcal{T} = \mathcal{K} = [0, 1]$ (other intervals only change constants) and assume for ease of formulation that the support of F contains at least k points. We will characterise the first-best mechanisms and give a tight characterisation of convergence of ex-ante average welfare.

Definition. *Let $F \colon \mathbb{R} \to [0, 1]$ be a square-integrable distribution and $X \sim F$. Then an $\mathcal{M}$-quantisation of F is a pair of functions $(f^{\downarrow}, f^{\uparrow})$, $f^{\downarrow} \colon \mathbb{R} \to \mathcal{M}$, $f^{\uparrow} \colon \mathcal{M} \to \mathbb{R}$. An $\mathcal{M}$-quantisation$(f^{\downarrow}, f^{\uparrow})$ is called* optimal *if it minimises* $\mathrm{MSE}(F, (f^{\downarrow}, f^{\uparrow})) \coloneqq \mathbb{E}[(X - f^{\uparrow}(f^{\downarrow}(X)))^2]$. *Call the minimiser* $\mathrm{MSE}^*(F)$.

We say that X is sent to *quantisation level* $f^{\downarrow}(X)$ with *representative point* $f^{\uparrow}(f^{\downarrow}(X))$.

An example of a quantisation is shown in Figure 1. The function $f^{\downarrow}$ can be seen as partitioning the positive real line into intervals via their preimages, here denoted by

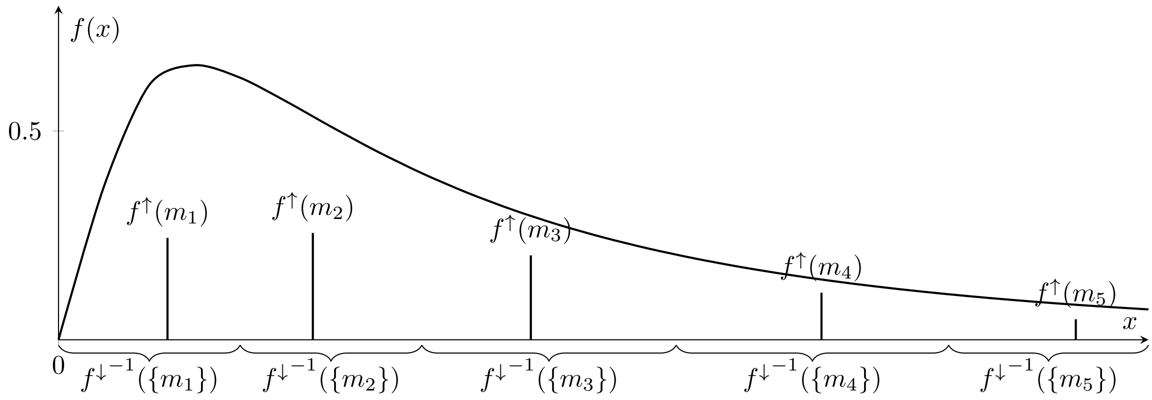

Figure 1: Example of an $\{m_1, m_2, \ldots, m_5\}$-quantisation.

braces. Values within an interval are all mapped to the same value m_i, that is mapped to the representative point $f^{\uparrow}(m_i)$. The following characterises optimal quantisations.

Proposition (Lloyd 1982, Eqn. (16) and (17))**.** *The optimal $\mathcal{M}$-quantisation satisfies*

$$f^{\downarrow}(x) = \arg\min_{m \in \mathcal{M}} (x - f^{\uparrow}(m))^2 \tag{3}$$

$$f^{\uparrow}(m) = \mathbb{E}_{X \sim F}[X | X \in (f^{\downarrow})^{-1}(\{m\})]. \tag{4}$$

The resulting quantisation is also called Lloyd-Max quantisation*.*

Equation (3) says that each value shall be mapped to the closest representative point of any quantisation level, equation (4) says that the representative point of a quantisation level shall be the centroid of all points that are mapped to this level.

The main result of this section uses quantisation to give a strong necessary condition for first-best mechanisms with restricted communication:

Theorem 1 (Characterisation: First-Best)**.** *Let F have optimal $\mathcal{M}$-quantisation $(f^{\uparrow}, f^{\downarrow})$. Then there is a first-best mechanism $g_{\text{first-best}}$ together with implementing strategies s_1, s_2, $\ldots$, s_n such that*

$$g_{\text{first-best}}(m_1, m_2, \ldots, m_n) = \frac{1}{n} \sum_{i=1}^{n} f^{\uparrow}(m_i), \qquad s_i(x) = f^{\downarrow}(x).$$

In other words, the same quantisation is applied to each agent's type separately and the reported representative points are averaged.

In the case of unrestricted communication, the best average welfare is obtained by the "mean mechanism", the social choice function $f \colon \mathcal{T}^n \to \mathcal{K}, (t_1, t_2, \ldots, t_n) \mapsto \frac{1}{n} \sum_{i=1}^{n} t_i$. It obtains an average welfare of $W(F,f) = -\frac{n-1}{n}\sigma^2$. Therefore, it is fair to consider the quantity $-\frac{n-1}{n}\sigma^2 - W(F,f) \geq 0$ and its convergence to zero in welfare analysis.

Proposition 2 (Welfare: First-Best)**.** *Let F have variance σ^2. Then*

$$\frac{1}{12nk^2} \leq -\frac{n-1}{n}\sigma^2 - W(F, g_{\text{first-best}}) \leq \frac{1}{4nk^2}. \tag{5}$$

Note that the loss due to quantisation is relatively minor: In both k and n, the loss that is incurred also in the communication-unrestricted case is of another order of magnitude than the loss due to the communication restriction.

Finally, we observe that the range of the first-best mechanism grows in both n and k.

Corollary 3 (Range: First-Best)**.** $|\operatorname{range} g_{\text{first-best}}| \geq (k-1)n + 1$ *and this is tight.*

3.2 Second-Best

In many situations, the first-best is not implementable in dominant-strategies. This occurs for $k \geq 3$ when it is possible for agents to exaggerate their preferences. As we are interested in implementable mechanisms, we continue to study second-best mechanisms—for tractability on the domain of single-peaked preferences.

We introduce a class that generalises generalised median voting schemes introduced by Moulin 1980 that are well-known in the literature on strategy-roof social choice.

Definition. *An* embedded generalised median voting rule *is a mechanism $g\colon \mathcal{M}^n \to \mathcal{K}$ if there is an injective function $\iota\colon \mathcal{M} \to \mathcal{K}$ (the* embedding*) and $\alpha_1, \alpha_2, \ldots, \alpha_{n-1} \in \operatorname{range}\iota$ (the* phantom ballots*) such that*

$$g\colon \mathcal{M}^n \to \mathcal{K}, (m_1, m_2, \ldots, m_n) \mapsto \operatorname{median}\{\iota(m_1), \iota(m_2), \ldots, \iota(m_n), \alpha_1, \alpha_2, \ldots, \alpha_{n-1}\}.$$

Figure 2 illustrates generalised median voting rules. As for the first-best mechanisms, the strategies s_i yield a partition of the type space that is illustrated by braces. There is a bijective mapping between the message space $\mathcal{M}$ and the range $\mathcal{F}$ of the mechanism g. The agents send messages m that are interpreted as ballots for the representative point $\iota(m)$ (upwards arrows). The median of the votes for the different values (black bullets), together with phantom ballots (grey bullets) that can only lie on representative points, is implemented (downwards arrow).

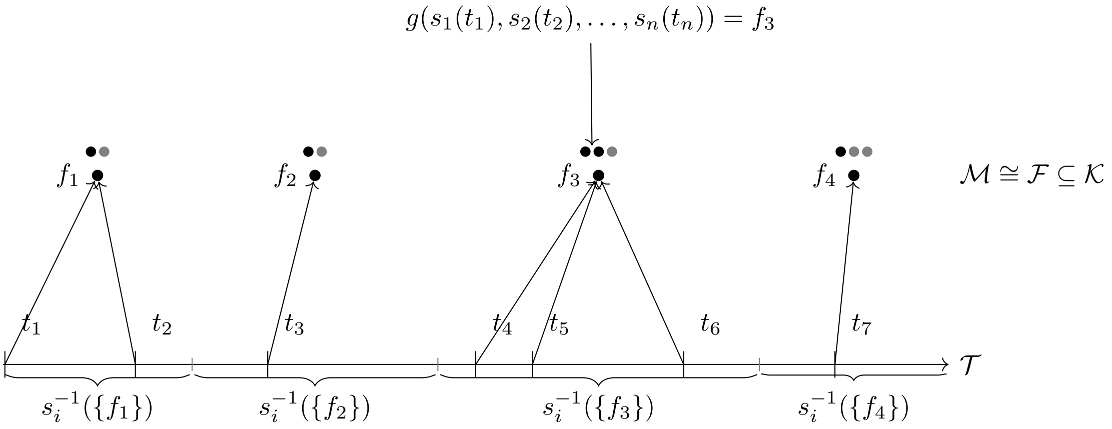

Figure 2: Embedded generalised median voting rules

The following is the main result of this section. It shows that a restriction on the communication of agents to the principal rules out all mechanisms but embedded generalised median voting rules, that are restricted in that they have a small range.

Theorem 4 (Characterisation: Second-Best). *Let $\mathcal{T}$ be the set of single-peaked preferences on $\mathcal{K}$. Then $g\colon \mathcal{T}^n \to \mathcal{K}$ is anonymous, non-wasteful and DIC implementable by surjective strategies if and only if it is an embedded generalised median voting rule.*

Corollary 5 (Range: Second-Best). $|\operatorname{range} g_{\text{second-best}}| \leq k$.

This theorem is surprising in several respects:
First, we observe that, endogenously, an embedding of the message set into the choice set emerges, whose computation is much easier to study than the original social choice optimisation problem.
Second, the theorem says that a bound on the agents' communication implies that the principal's communication can be limited, too: Only k values (positions of the representative points) and k integers of size smaller than n (numbers of phantom ballots on each representative point) have to be communicated.
Finally, Theorem 4 can be seen as evidence for the prevalence of parties in representative systems. Even though *de lege* in many constitutions only representatives are voted upon and parties are not mentioned (e.g. German constitution, art. 38), they *de facto* have an important role as representative ideological positions. In light of Theorem 4, parties can be seen as representative points of an embedded median voting rule.

To close this section, we show that the first- and second-best case differs with respect to welfare convergence. The difference in welfare of any embedded generalised median voting rule compared to an incentive compatible mechanism in a communication-unrestricted setting goes to zero slower than the difference of the communication-restricted to the communication-*un*restricted first-best's welfare (the latter difference has been $O(k^{-2})$). This can be seen as an indication that the welfare implications of restricted communication are stronger if incentives are considered.

Proposition 6 (Welfare: Second-Best)**.** *Assume quadratic preferences, $n = 2$ and F to be the uniform distribution on $[0, 1]$. Then for any unanimous, anonymous, strategy-proof mechanism g (in a model without communication restriction) and any sequence $(g_k)_{k\in\mathbb{N}}$ of embedded generalised median voting rules $g_k\colon \mathcal{M}_k^n \to \mathcal{K}$, $|\mathcal{M}_k| = k$,*

$$W(F, g) - W(F, g_k) \in \Omega(k^{-1}).$$

4 A More General Preference Restriction

The Sections 3.1 and 3.2 are disconnected in the sense that the domain of quadratic preferences is strictly smaller than the set of single-peaked preferences. One should aim for results on preference domains that are smaller than the set of single-peaked preferences, as some reports of single-peaked preferences might be counter-factual in application domains (e.g. increasing preference for higher taxes if the most preferred tax rate is a low one). We define a preference restriction allowing for smaller domains

Definition. *$\mathcal{T}$ and $\mathcal{K}$ together form a RST domain if*

(a) $\mathcal{T}$ is a regular domain*: For any $k \in \mathcal{K}$ there is $t \in \mathcal{T}$ for which $k = \tau_{\mathcal{K}}(t)$.*

(b) $\mathcal{T}$ is single-crossing*: If $k < k' \in \mathcal{K}$, $t < t' \in \mathcal{T}$ $k \preceq^t k'$, then $k \preceq^{t'} k'$.*

(c) $\mathcal{T}$ is tops-connected: For any finite $A \subseteq \mathcal{K}$, and adjacent[1] $x, y \in A$ there is $t_A \in \mathcal{T}_A$ such that $\tau_A(t_A) = x$ and $\tau_{A\setminus\{x\}}(t_A) = y$, i.e. x is t_A's first, y its second choice.

The RST preference domain is sufficiently general: It not only includes the quadratic model of preferences, but also the following model adapted from Romer et al. 1975, that models voting on linear tax schedules: Preferences satisfy the *linear* preference restriction if they are induced by utility functions $u^x_{\text{lin.}}(t) = a(x)+b(x)t$, where $a\colon [0, 1] \to \mathbb{R}$ is strictly decreasing and $b\colon [0, 1] \to \mathbb{R}_{\geq 0}$ is strictly increasing and non-negative.[2]. Unfortunately,

[1] $x, y \in A$ are adjacent if there is no $z \in A$ such that $x < z < y$ or $x > y > z$.

[2] One needs an additional property to ensure that RST holds: $(u^x)_{x\in\mathbb{R}}$ should form the sub-differentials of a convex function. It can be seen that this is equivalent to the assertion that if agents choose their most preferred tax schedule, the after-tax income should grow super-linearly in the productivity type, which is fair to assume.

we are unable to prove that for RST domains, Theorem 4 holds more generally, but what we can prove using recent results from Achuthankutty and Roy 2018, property B', comes close to what is needed (property B). For a preference domain $\mathcal{T}$ and a choice set $\mathcal{K}$, define: Let $\mathcal{T}$ be a preference domain. Define the following axioms:

Property A For any subset $\mathcal{T}$ of A, $\tau_A \colon \mathcal{T}_A \to A$ is monotone and surjective.

Property B For any finite $A \subseteq \mathcal{K}$, a social choice function $\mathcal{T}_A^n \to A$ is strategy-proof, surjective and anonymous if and only if it is a generalised median voting rule.

Property B' For any finite $A \subseteq \mathcal{K}$, a social choice function $\mathcal{T}_A^n \to A$ is strategy-proof, unanimous and anonymous if and only if it is an generalised median voting rule.

Proposition (RST domains: property A)**.** *RST implies A and B'.*

Theorem 7 (Characterisation: Second-Best, RST)**.** *Let $\mathcal{T}$ be a preference domain that satisfies properties A and B. Then $g \colon \mathcal{T}^n \to \mathcal{K}$ is anonymous, non-wasteful and DIC implementable by surjective strategies if and only if it is an embedded generalised median voting rule.*

We conjecture that RST domains satisfy property B, too, implying, among others, that the linear and quadratic preference domains satisfy the characterisation Theorem 7. As this result is a result purely on strategy-proof social choice, Theorem 7 shows that this field of research has implications also for communication-restricted settings.

5 Conclusion

The present thesis studied information-constrained voting. The main difference to other models of voting is that the assumption of abundant capacity of communication from the agents to the principal is dropped. We characterise first- and second-best mechanisms, analyse welfare convergence and show that the range of incentive compatible mechanisms is smaller than first-best mechanisms.

Further directions include the study of BIC models of voting with restricted communication, that are likely more complex than the present study in a DIC setting, the derivation of optimality criteria that allow for the computation of representative points and the study of higher-dimensional (spatial) models of voting with restricted communication.

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