EXPLOITING THE COMPUTATIONAL BURDEN: MODEL

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ABSTRACT. We consider a static screening problem where agents have heterogenous capabilities to access the optimal menu.

1. Model

We consider a static screening model with limited menu access from the agents. The seller has an absolutely continuous, regular prior on agent types $\theta \sim F$, $\theta \in [0,1]$.[1] In addition, we assume that F's density is bounded by C [2]. In addition, each type has a deterministic and finite *patience* $m(\theta)\colon [0,1] \to \{1,2,\ldots,M\}$.

The seller designs a *menu measure* μ on pairs $(q,t) \in [0,1]\times[0,1]$. An agent with type θ draws a (random) family $\tilde{\mathcal{M}}_{\theta(m)} = \{(q_i,t_i)\}_{i=1}^m$ of $m(\theta)$ independent samples according to μ, (q_1,t_1), (q_2,t_2), $\ldots$, $(q_m,t_m) \sim \mu$. The buyer chooses $(q(\theta),t(\theta)) \in \mathcal{M}_{m(\theta)} \coloneqq \tilde{\mathcal{M}}_{m(\theta)} \cup \{(0,0)\}$ to maximise a quasi-linear utility function

$$u_\theta(q,t) = q\theta - t.$$

We suppress the dependence of $\mathcal{M}_{m(\theta)}$ and $(t(\theta),q(\theta))$ on μ and m for notational convenience. The seller wishes to choose the menu measure μ to maximise revenue

$$\mathcal{R}_{F,m} = \sum_{\mu} \mathbb{E}[t(\theta)].$$

As a benchmark, we compare the revenue received in this setting to the static screening model of [Mye82]. Agent types are distributed according to a distribution F that admits a regular density f. In contrast to our model, the seller here designs a *set* $\mathcal{M} \subseteq [0,1]\times[0,1]$. An agent with type θ chooses $(q(\theta),t(\theta)) \in \mathcal{M}$ to maximise his quasi-linear utility function (1). The seller wishes to maximise revenue

$$\mathcal{R}_F \coloneqq \sup_{\mu} \mathbb{E}[t(\theta)].$$

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[1] We do not consider ironing, [Mye82] in this note.

[2] This assumption is implied by the assumption of continuous density with bounded derivative in [DDT179, KM19].

For a distribution F and a function m, we call

$$\mathcal{R}_{F,m} - \mathcal{R}_F$$

the *profit from the computational burden* or *profit from impatience*, where we will use the second name more frequently. We will regularly use $\delta_\bullet$ to denote a point mass/a Dirac measure and $[k] := \{1, 2, \dots, k]\}$ to denote the first k positive natural numbers.

A few comments on our assumptions are in place.

First, observe that the lack of control of the seller of the sampling process does not allow for a revelation principle. Indeed, a revelation principle would rule out any positive profits from impatience: If there was a revelation principle, then the direct revelation mechanisms, which are the most flexible ones among those in which agents report their types, would give the maximal revenue. Therefore, only *ad hoc* relaxations such as ε-incentive compatibility or approaches like ours can allow for strictly higher revenue beyond direct mechanisms.

The assumption that each agent samples only a finite number of menu items might be challenged. On the one hand, arbitrarily accurate sampling cannot be expected from real-world agents. On the other hand, allowing for infinite patience gives even problems with existence.[3]

A third assumption is that patience determinstically depends on types is not a crucial one and proofs can be adapted to the case of an arbitrary correlation structure of types and the levels of patience.[4] For ease of exposition, we chose to present the deterministic model.

Proposition 1. *If some of the agents can sample infinitely, then the set of menu measures μ achieving at least revenue k is not necessarily closed, hence the mapping from menu measures is not in general continuous.*

The proof is by an example, which at the same time gives an insight on how a strictly positive profit from impatience can be achieved.

Example 1 (Non-Existence)**.** *Let $F = \mathrm{Unif}_{[0,1]}$. In this case, for the benchmark model, it is well-known that a posted price of $\frac{1}{2}$ and a revenue of $\frac{1}{4}$ are optimal. Assume the following patience for agents:*

$$m(\theta) = \begin{cases} \infty & \text{if } \theta \leq 1 - \frac{1}{k} \\ 1 & \text{else.} \end{cases},$$

where $k > 2$. In this example, the agents that are most willing to buy have a low patience, whereas all other agents have infinite patience. We now

[3]Our model assumes uniformly bounded patience. Example 1 below assumes infinite patience. We do not study the case of finite, but not uniformly bounded patience, as we expect similar behavior as for infinite patience if not assuming additional structure on m, in particular non-existence of optimal menu measures.

[4]Indeed, this would involve introducing several different types that are utility-identical, but have different patience levels. Similar techniques as used here can be applied.

construct a family of measures that yield higher and higher revenue above the profit, but whose limit does give as much revenue.

Consider the menu that offers with ε *probability the optimal posted price and full allocation, and otherwise a price that would exactly let impatient agents buy,*

$$\mu_\varepsilon = \varepsilon\delta_{(1,\frac{1}{2})} + (1-\varepsilon)\delta_{(1,1-\frac{1}{k})},$$

Agents $\theta \in [0, \frac{1}{2}]$ *and* $\theta \in [\frac{1}{2}, 1-\frac{1}{k}]$ *each can choose from the menu*

$$\left\{(0,0), \left(1, \frac{1}{2}\right), \left(1, 1-\frac{1}{k}\right)\right\}$$

and choose, respectively, $(0,0)$ *and* $(1, \frac{1}{2})$*, yielding a total revenue of* $\frac{1}{2}(1 - \frac{1}{k} - \frac{1}{2})$ *from these types. Agents* $\theta \in [1-\frac{1}{k}, 1]$ *sample one element from* μ *and their menu is*

$$\begin{cases} \{(0,0),(1,\frac{1}{2})\} & \textit{with probability } \varepsilon \\ \{(0,0),(1,1-\frac{1}{k})\} & \textit{with probability } 1-\varepsilon \end{cases}$$

and they choose, respectively, $(1, \frac{1}{2})$ *and* $(1, 1-\frac{1}{k})$*. This yields a total revenue from these types of*

$$\frac{1}{k}\left(\frac{\varepsilon}{2} + \left(1-\frac{1}{k}\right)(1-\varepsilon)\right).$$

This gives a total revenue of

$$\frac{1}{2}\left(1-\frac{1}{k}-\frac{1}{2}\right) + \frac{1}{k}\left(\frac{\varepsilon}{2} + \left(1-\frac{1}{k}\right)(1-\varepsilon)\right) = \frac{1}{4} + \frac{5-\varepsilon}{2k} - \frac{1-\varepsilon}{k^2}.$$

which is, by assumption $k \geq 2$ *strictly greater than* $\frac{1}{2}$ *and decreasing in* ε*. Therefore, the set* $\{\mu_\varepsilon | \varepsilon \in (0,1)\}$ *is contained in an upper contour set of the function assigning menu measures to revenue that does not contain the optimal posted-price menu* $\delta_{(1,\frac{1}{2})}$*. But, as a weak convergence of measures,*

$$\mu_\varepsilon \xrightarrow{\varepsilon \to 0} \delta_{(1,\frac{1}{2})},$$

which shows that the upper contour set is not closed.

This example already shows two features of the model that will come out in the rest of the article.

First, the finiteness of sampling itself can be seen as a regularising tool. Indeed, if a positive mass of the agents in $[\frac{1}{2}, 1-\frac{1}{k}]$ had a finite patience, there would be a (small) probability that they do not see the menu item $(1, \frac{1}{2})$, which would mean a loss for the auctioneer at very small values of ε. Therefore, finite sampling prevents the emergence of purely promotional offers that an agent can never use.

A second important feature in this example was the non-monotonicity of patience. Indeed, as we will show in the next section, if m is almost everywhere equal to a monotone function, there will be no profit from impatience.

References

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